NMR C-NOT gate through the Aharanov-Anandan phase shift

2001 J. Phys. A: Math. Gen. 34 L631

(http://iopscience.iop.org/0305-4470/34/44/105)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **34** (2001) L631–L634 PII: S0305-4470(01)25189-0

LETTER TO THE EDITOR

NMR C-NOT gate through the Aharanov–Anandan phase shift

Xiangbin Wang and Keiji Matsumoto

Imai Quantum Computation and Information Project, ERATO, Japan Science and Technology Corporation, Daini Hongo White Building 201, 5-28-3 Hongo, Bunkyo, Tokyo 113-0033, Japan

E-mail: wang@qci.jst.go.jp and keiji@qci.jst.go.jp

Received 22 May 2001
Published 26 October 2001
Online at stacks.iop.org/JPhysA/34/L631

Abstract
Recently, it has been proposed to perform quantum computation through the Berry phase (adiabatic cyclic geometric phase) shift with NMR. This geometric quantum gate will hopefully be fault tolerant to certain types of error because of the geometric property of the Berry phase. Here we give a scheme to realize the NMR C-NOT gate through the Aharonov–Anandan phase (non-adiabatic cyclic phase) shift on the dynamic phase free evolution loop. In our scheme, the gate is run non-adiabatically, thus it is less affected by the decoherence. Moreover, in the scheme we have chosen the zero dynamic phase time evolution loop in obtaining the geometric phase shift; we need not perform any extra operation to cancel the dynamic phase.

PACS numbers: 03.67.Lx, 03.65.Vf

It has been shown that a quantum computer, if available, can perform certain tasks much more efficiently than a classical Turing machine [1–3]. The realization of the basic constituent of a quantum computer, namely a fault-tolerant quantum logic gate, is a central issue. It has been shown that, with the single-qubit rotation operation and the C-NOT gate, one can in principle realize arbitrary quantum computation [4].

Geometric phase [5–7] plays an important role in quantum interferometry and many other disciplines. For the two-level system, geometric phase is equal to half of the solid angle subtended by the area in the Bloch sphere enclosed by the closed evolution loop of the eigenstate. Recently [8], it was proposed to make a fault-tolerant C-NOT gate using Berry's phase [6], i.e. adiabatic and cyclic geometric phase. A similar idea was then developed to an asymmetric SQUID system [9]. However, the proposal [8] relies on the adiabatic operations. As has been shown in [8], due to its geometric property, geometric phase is fault tolerant to certain types of operational error, so the proposals of quantum computation through conditional geometric phase shift are potentially important in the future implementation of a fault-tolerant quantum computer. However, one should overcome two drawbacks in the previous suggestions [8,9] in performing geometric quantum computation. One is the adiabatic condition, which makes such a gate not practical. The decoherence effects may distort the laboratory observations seriously. The experimental results with systematic errors were obtained on NMR [8]. It is also reported that the distortion is increased seriously with the faster running speed. The other drawback is the extra operation to eliminate the dynamic phase. This extra operation weakens the fault-tolerant property [10]. In this Letter, we give a simple scheme to realize the geometric C-NOT gate non-adiabatically on the 0 dynamic phase evolution curve with NMR. By this scheme, the above-mentioned two drawbacks of previous suggestions are removed and the realization can be achieved faster and more easily. We believe our scheme has led to the idea of a geometric C-NOT gate much more practical than before.

Geometric phase also exists in non-adiabatic processes. It was shown by Aharonov and Anandan [7] that the geometric phase is only dependent on the area enclosed by the loop of the state on the Bloch sphere. In the non-adiabatic case, the path of the state evolution in general is different from the path of the parameters in the Hamiltonian. The external field need not always follow the evolution path of the state like that in the adiabatic case. It is therefore possible to let the external field be perpendicular to the evolution path instantaneously so that there is no dynamic phase involved in the whole process.

The detection of AA phase was successfully performed many years ago [11]. In particular, one may select the geodesic evolution loop so that no dynamic phase is involved in the interference result. It should be interesting to develop a non-adiabatic and dynamic phase free scheme to realize the two-qubit C-NOT gate through geometric phase shift with NMR [8]. The scheme [11] for non-adiabatic detection of geometric phase for a single qubit can be easily developed for the two-qubit system to make a C-NOT gate with NMR.

Consider the interacting nucleus spin pair (spin a and spin b) in the NMR quantum computation [12–15]. For simplicity we call them qubit a and qubit b, respectively. We shall use subscripts a and b to indicate the corresponding qubits. If there is no horizontal field the Hamiltonian for the two-bit system is $H_i = \frac{1}{2}(\omega_a\sigma_{za} + \omega_b\sigma_{zb} + J\sigma_{za}\cdot\sigma_{zb})$, where $\omega_{a,b}$ is the resonance frequency for spin a, b respectively in a very high $+z$ direction static magnetic field (e.g. ω_a can be 500 MHz [8]), J is the interaction constant between nuclei and $\sigma_{za} = \sigma_{zb} = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The Hamiltonian for spin a in the rotational framework of rotation speed $\omega'_a = \omega_a - J$ is

$$H_a = R'H'R'^{-1} + \mathrm{i}(\partial R'/\partial t)R'^{-1} = \tfrac{1}{2}(\omega_a - \omega'_a \pm J)\sigma_z \tag{1}$$

and $R' = \mathrm{e}^{\mathrm{i}\omega'_a\sigma_z t/2}$. Note here that H_a is dependent on the state of spin b through $\pm J$. Explicitly, it is $\frac{1}{2}\cdot(2J)\sigma_z$ if the state of qubit b is $|\psi_b\rangle = |\uparrow\rangle$, and *it is* zero if the state of qubit b is $|\psi_b\rangle = |\downarrow\rangle$.

Suppose we have chosen ω_a obviously different from ω_b. This means that while we take operations on qubit a, the state of qubit b is (almost) not affected.

We first rotate qubit a around the x axis through angle $-\theta$ (see figure 1). This operation is denoted by $(-\theta)^x$. Note that $|\omega_b - \omega'_a|$ is much larger than J, so the state of qubit b is (almost) not affected by any operation on qubit a in the whole process. The interaction Hamiltonian will create an evolution path on the geodesic circle ABC (see figure 1). After time $\tau = \pi/(2J)$, we rotate qubit a around the x axis through another angle $-(\pi - 2\theta)$. Again wait for a time τ. Then rotate qubit a around the x axis through angle $\pi - \theta$ to return the Bloch sphere to the original one. In short, the above scheme can be expressed as

$$\hat{S} = (-\theta)^x \longrightarrow \tau \longrightarrow [-(\pi - 2\theta)]^x \longrightarrow \tau \longrightarrow (\pi - \theta)^x. \tag{2}$$

In the scheme we have used the rotation operation on qubit a, around the x axis. This can be done by an rf pulse.

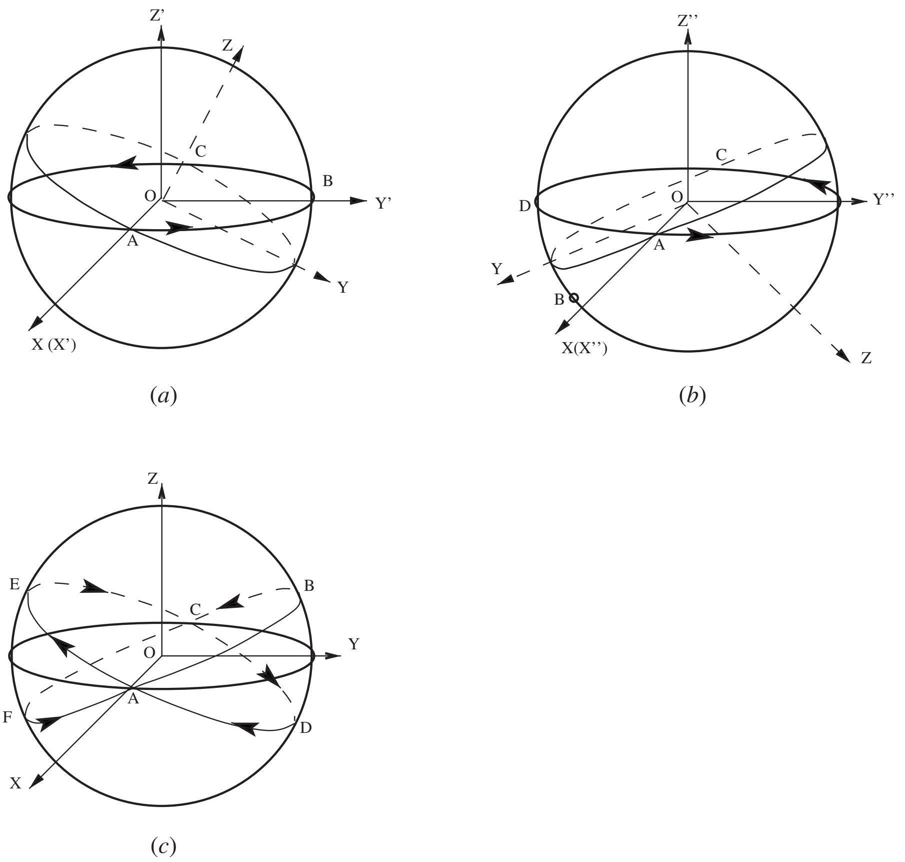

Figure 1. Non-adiabatic conditional geometric phase shift acquired through zero dynamic phase evolution path. These are pictures for the time evolution on the Bloch sphere of qubit a in the case where qubit b is $|\uparrow\rangle$. (a) shows that after the Bloch sphere is rotated around the x axis through angle $-\theta$, the interaction Hamiltonian will rotate the Bloch sphere around the z' axis. At the time it completes π rotation, i.e. $\tau = \pi/(2J)$, we rotate the Bloch sphere around the x axis again through an angle of $-(\pi - 2\theta)$, then we obtain (b). In (b) the sphere is rotated around the z'' axis by the interaction Hamiltonian. Note that point B in (b) has changed its position now. The geodesic curve CBA is not drawn in (b). After time τ we rotate qubit a around the x axis through an angle of π–θ. (c) The whole evolution path on the Bloch sphere. Point A evolves along closed curve ABCDA; a geometric phase $\gamma = -2\theta$ is acquired. Point C evolves along the loop CFAEC; a geometric phase $-\gamma = 2\theta$ is acquired. These are pictures for the time evolution on the Bloch sphere of qubit a *only* in the case where qubit b is $|\uparrow\rangle$. If qubit b is $|\downarrow\rangle$, after the operation, qubit a returns to its initial state exactly.

After the above operation, if qubit b is in state $|\uparrow\rangle$, an evolution path of ABCDA or CFAEC on the Bloch sphere is produced for qubit a; if qubit b is in state $|\downarrow\rangle$, nothing happens to qubit a. This is equivalent to saying the time evolution operator has the property $U(2\tau)|\pm\rangle = \mathrm{e}^{\pm \mathrm{i}\gamma(\theta)}|\pm\rangle$ if qubit b is up and $U(2\tau) = 1$ if qubit b is down. Here states $|\pm\rangle$ correspond to points A and C respectively in the Bloch sphere. $\gamma(\theta)$ is the geometric phase acquired for initial state $|+\rangle$ (point A in figure 1). It is the half solid angle subtended by the area ABCDA and $\gamma(\theta) = -2\theta$. In the basis of $|\uparrow\rangle$ and $|\downarrow\rangle$ (eigenstate of σ_z), if qubit b is up we have the following time evolution formula for qubit a:

$$U(2\tau)\begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix} = \begin{pmatrix} \cos\gamma(\theta) & \mathrm{i}\sin\gamma(\theta) \\ \mathrm{i}\sin\gamma(\theta) & \cos\gamma(\theta) \end{pmatrix} \begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}. \tag{3}$$

Note that if qubit b is down there is no change to qubit a, in any basis. We see $|\gamma(\theta)| = \pi/2$ makes a C-NOT gate here (see figure 1). This corresponds to $\theta = \pi/4$. Note here that the state of qubit a itself in general does not undergo a cyclic evolution. However, the evolution path of qubit a is completely determined by γ, which is the AA phase of state $|+\rangle$.

In summary we have a scheme that can be used to make a C-NOT gate through pure geometric phase, which is fault tolerant to certain types of error [8]. This γ is fault tolerant to those errors which do not change the area enclosed by the evolution loop of states $|\pm\rangle$. Here we need not take any extra action to remove the dynamic phase [8,9]. Moreover, in our scheme the operation is performed non-adiabatically, so the running speed of our geometric gate is at the same level as the normal gate. We believe our scheme has led the idea of a geometric C-NOT gate [8] much closer to practical use.

We thank Professor Imai for support. WXB thanks Professor A Ekert (Oxford) for fruitful discussions.

References

[1] Lloyd S 1993 *Science* **261** 1569
[2] Bennett C H 1995 *Phys. Today* **48** 24
[3] DiVincenzo D P 1995 *Science* **269** 255
[4] Deutsch D, Barenco A and Ekert A 1995 *Proc. R. Soc.* A **449** 669–77
[5] Pancharatnam S 1956 *Proc. India Acad. Sci.* A **44** 247
[6] Berry M V 1984 *Proc. R. Soc.* A **392** 45
[7] Aharonov Y and Anandan J 1987 *Phys. Rev. Lett.* **58** 1593
[8] Jones J A, Vedral V, Ekert A and Castagnoli G 2000 *Nature* **403** 869
[9] Falcl G, Fazio R, Palma G M, Siewert J and Verdal V 2000 *Nature* **407** 355
[10] Alexandre B and Andre-Marie S M 2001 *Preprint* quant/ph-0105006
[11] Suter D, Muller K T and Pines A 1988 *Phys. Rev. Lett.* **60** 1216
[12] Cory D G, Fahmy A F and Havel T F 1997 *Proc. Natl Acad. Sci. USA* **94** 1634–9
[13] Gershenfeld N A and Chung I L 1997 *Science* **275** 350–6
[14] Jones J A and Mosca M 1998 *J. Chem. Phys.* **109** 1648–53
[15] Jones J A, Hansen R H and Mosca M 1998 *J. Magn. Reson.* **135** 353–60